

 **Computational Materials Physics**

Center for Molecular Modeling

Department of Materials Science and Engineering

## bulk modulus and pressure

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<http://molmod.ugent.be>  
<http://www.ugent.be/ea/dmse/en>  
my talks on YouTube: <http://goo.gl/P2b1Hs>

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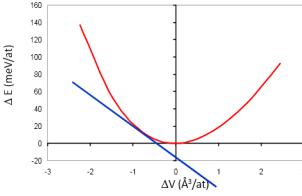


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Bulk modulus  $B$  = pressure change required to realize a given relative volume change, at a given volume.

$$\begin{aligned} B_0 &= -\frac{\Delta p}{\frac{\Delta V}{V_0}} & dU(S, V) &= TdS - pdV \\ &= -\frac{p - 0}{\frac{V - V_0}{V_0}} & \frac{\partial U}{\partial V}(S, V) &= -p \\ &= -V_0 \frac{\Delta p}{\Delta V} & & \\ &= -V_0 \frac{\partial p}{\partial V} & & \\ &= V_0 \frac{\partial^2 U}{\partial V^2} & & \end{aligned}$$



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Bulk modulus  $B$  = pressure change required to realize a given relative volume change, at a given volume.

$$\begin{aligned} B_0 &= -\frac{\Delta p}{\frac{\Delta V}{V_0}} \\ \text{at equilibrium} &= -\frac{p - 0}{\frac{V - V_0}{V_0}} \\ &= -V_0 \frac{\Delta p}{\Delta V} \\ &= -V_0 \frac{\partial p}{\partial V} \\ &= V_0 \frac{\partial^2 U}{\partial V^2} \end{aligned}$$

B can be defined at any other volume (or pressure) too  
→ B is pressure dependent

$$\begin{aligned} B(p) &= B(0) + B'p + \frac{1}{2}B''p^2 + \dots \\ &= B_0 + B_1p + \frac{1}{2}B_2p^2 + \dots \end{aligned}$$

at equilibrium

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Other equations of state:

**Birch-Murnaghan** (valid up to 10% volume change):

[http://en.wikipedia.org/wiki/Birch%E2%80%93Murnaghan\\_equation\\_of\\_state](http://en.wikipedia.org/wiki/Birch%E2%80%93Murnaghan_equation_of_state)

$$P(V) = \frac{3B_0}{2} \left[ \left( \frac{V_0}{V} \right)^{\frac{7}{3}} - \left( \frac{V_0}{V} \right)^{\frac{5}{3}} \right] \left\{ 1 + \frac{3}{4} (B_1 - 4) \left[ \left( \frac{V_0}{V} \right)^{\frac{2}{3}} - 1 \right] \right\}$$

$$E(V) = E_0 + \frac{9V_0 B_0}{16} \left\{ \left[ \left( \frac{V_0}{V} \right)^{\frac{2}{3}} - 1 \right]^3 B_1 + \left[ \left( \frac{V_0}{V} \right)^{\frac{2}{3}} - 1 \right]^2 \left[ 6 - 4 \left( \frac{V_0}{V} \right)^{\frac{2}{3}} \right] \right\}$$

From Physical Review B 52 (1995) 8064 ("eos2"):

$$E = a + bV^{-1/3} + cV^{-2/3} + dV^{-1}$$

Useful paper: Ronggang 2010 (<http://stacks.iop.org/PhysScr/82/045014>)

Comprehensive yet accessible book: Anderson 1994

([http://books.google.be/books/about/Equations\\_of\\_State\\_for\\_Solids\\_in\\_Geophys.html?id=qSqiz2NK7TIC](http://books.google.be/books/about/Equations_of_State_for_Solids_in_Geophys.html?id=qSqiz2NK7TIC))

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accuracy of  $V_0$ ,  $B_0$  and  $B_1$

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