



Center for  
Molecular  
Modeling

## Computational Materials Physics



Department of  
Materials Science  
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what are elastic constants  
used for ?

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<http://molmod.ugent.be>  
<http://www.ugent.be/ea/dmse/en>  
my talks on Youtube: <http://goo.gl/P2b1Hs>

Several so-called "elastic moduli" can be derived from the elastic constant.

For cubic crystals:

Bulk modulus B: 
$$B = \frac{c_{11} + 2 c_{12}}{3}$$

Shear modulus G: 
$$G = \frac{c_{11} - c_{12} + 3 c_{44}}{5}$$

Young's modulus E: 
$$E = \frac{9BG}{3B + G}$$

micro-hardness Hv: 
$$H_v = (1 - 2 \nu) E / [6 (1 + \nu)]$$
  
with  $\nu = (3 B - 2 G) / [2 (3 B + G)]$

Speed of sound

Longitudinal: 
$$v = \sqrt{\frac{B + \frac{4}{3}G}{\rho}}$$

Shear: 
$$v = \sqrt{\frac{G}{\rho}}$$

Typical agreement with experiment (here for fcc Cu) :

	Our calculations	Experiment
Lattice constant [Å]	3.64	3.61 [1]
$C_{11}$ [GPa]	176.7	176.20 [2]
$C_{12}$ [GPa]	122.4	124.94 [2]
$C_{44}$ [GPa]	78.0	81.77 [2]
Speed of sound [m/s]	4403	4439.5 [2]
Melting temperature [K]	1577	1356 [3]
Bulk modulus [GPa]	137.1	142.0 [2]
Shear modulus [GPa]	57.66	59.3 [2]
Young's modulus [GPa]	151.7	156.2 [2]
Poisson's Ratio	31.9%	31.7% [2]

(Taken from an exam by Yves Beghein, Jan Jaeken and Steven Vandenbrande)

#### Born stability criteria:

Eigenvalues of the C-tensor must be positive

Example for the cubic case:

$$\begin{aligned} C_{11} &> 0 \\ C_{44} &> 0 \\ C_{11} - C_{12} &> 0 \\ C_{11} + 2C_{12} &> 0 \end{aligned}$$

Even if  $E_{\text{coh}}$  is positive (energetically stable), the material might not be mechanically stable

Up to now, we discussed elastic constants at  $p=0$ . Just as the bulk modulus, they can be generalized to any pressure.